Busbridge Junior School
Calculations Guidance

Contents

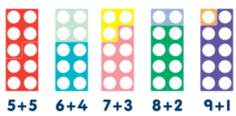
A flow chart, summarising the progression in calculation, can be found in the accompanying document "Calculations Guidance Annex".

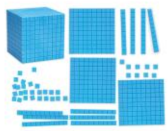
Introduction
Curriculum 2014 is based on the principles of **fluency** (rapid and accurate recall and sound conceptual understanding), **reasoning** (the ability to develop a line of inquiry, generalise and justify/prove their solution using mathematical language) and **solving problems** with an ability to break problems down into simple steps.

The ability to use appropriate written methods to add, subtract, multiply and divide, are fundamental to achieving these goals and underpin a sound knowledge of mathematics. This document sets out the progression of methods taught in our school to assist children in developing these skills.

The guidance reflects standards that we would expect the majority of children to achieve at various stages of their school life. However, the transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time. For those children requiring greater challenge, complete mastery of a concept will support deeper learning and an ability to manage more complex problems, rather than rapidly progressing through the stages.

The progression outlined in this guidance is aimed at providing children with an in-depth knowledge of the four operations, helping them to identify the relationships between them, and ways in which they relate to other mathematical concepts. It is essential that children develop an understanding of how each operation works, as opposed to merely learning methods by rote. The use of representational equipment (such as Cuisenaire rods and Numicon) and images (such as arrays and number lines) greatly assist in reinforcing and deepening such understanding.

Cuisinaire Rods  Numicon 

Dienes 

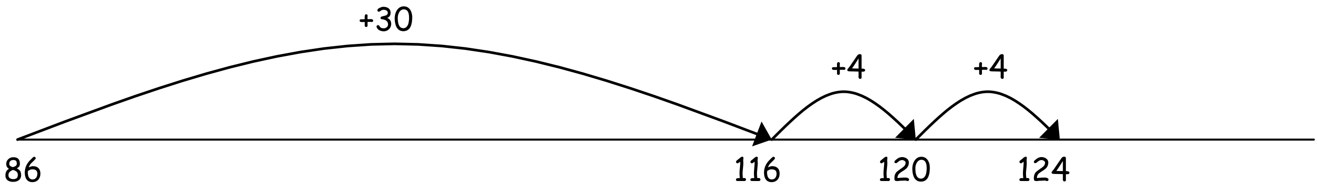
ADDITION

Y3

Children should be familiar with using empty number lines for addition. They will continue to use them with increasingly large numbers, bridging through tens and then hundreds and including compensation where appropriate.

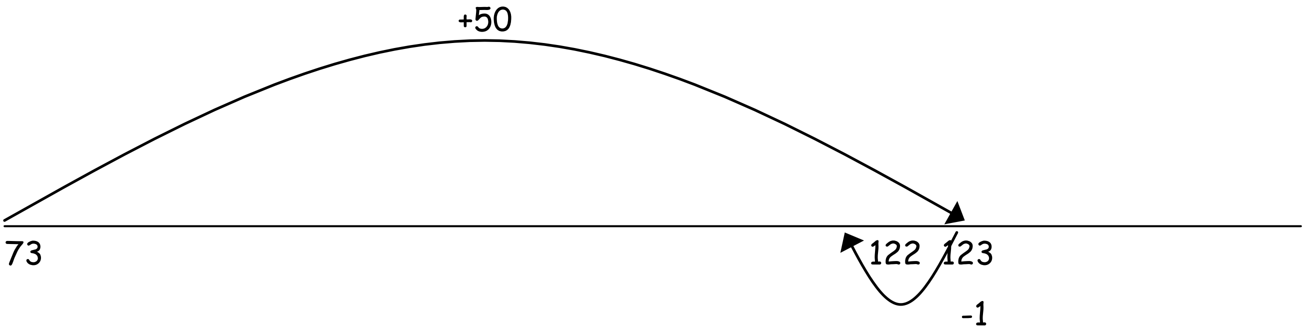
✓ Count on from the largest number irrespective of the order of the calculation.

38 + 86 = 124



✓ Compensation

49 + 73 = 122



Children will subsequently be introduced to partitioning two digit numbers, in order to add them together:

43 + 24	**37 + 46**	**56 + 62**
40 + 20 = 60	30 + 40 = 70	50 + 60 = 110
3 + 4 = 7	7 + 6 = 13	6 + 2 = 12
43 + 24 = 67	37 + 46 = 83	56 + 62 = 122

From this, children will begin to carry below (or above) the line.

43	37	56
+ 24	+ 46	+ 62
67	73	122
	1	1

Y4

Children will begin to carry below (or above) the line with numbers up to four digits.

```
  625        783        367
+  48      +  42      + 985
  673        825       1352
   1          1        111
```

Using similar methods, children will:

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;*
- ✓ *know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.*

Y5

Children should extend the carrying method to numbers with at least four digits and decimals.

```
   587       3587
 + 475     +  675
  1062       4262
   1 1       1 1 1
```

Using similar methods, children will:

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to three digits and the same number of decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m – 280 cm.*

Y6

Children should extend the carrying method to number with any number of digits.

```
   7648       6584          42
 + 1486     + 5848        6432
   9134      12432         786
   1 11      1 11            3
                        + 4681
                         11944
                          121
```

Using similar methods, children will

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 401.2 + 26.85 + 0.71.*

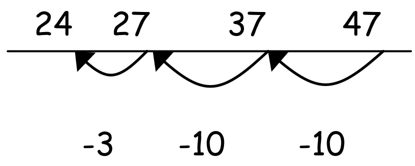
SUBTRACTION

Y3

Children will be familiar with using empty number lines for subtraction by counting back and counting on (see examples below). They will continue to use empty number lines with increasingly large numbers, bridging through tens and hundreds, as appropriate.

Counting back method

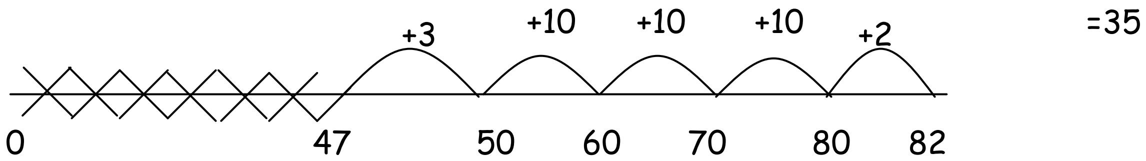
47 – 23 = 24



Counting on method

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

82 - 47



Partitioning and decomposition

Initially, the children will be taught using examples that do not need the children to include decomposition (i.e. the numbers being subtracted on the lower row are smaller than those above).

This process should be demonstrated using arrow cards to show the partitioning and base 10 materials (such as Dienes blocks) to show the decomposition of the number.

This method can be demonstrated by the teacher as means of introduction

$$\begin{array}{r} 89 \\ -\ 57 \\ \hline \end{array} \quad = \quad \begin{array}{rcr} 80 & + & 9 \\ 50 & + & 7 \\ \hline 30 & + & 2 \end{array} = 32$$

Children would write

$$\begin{array}{r} 89 \\ -\ 57 \\ \hline \mathbf{32} \end{array}$$

To introduce decomposition, this will be used as a demonstration method by the teacher

$$\begin{array}{llrcr} \text{Step 1} & & 70 & + & 1 \\ & - & \underline{40} & \underline{+} & \underline{6} \\ \end{array}$$

> The calculation should be read as e.g. take 6 from 1.

$$\begin{array}{llrcrcl} \text{Step 2} & & 60 & + & 11 & & \\ & - & \underline{40} & \underline{+} & \underline{6} & & \\ & & 20 & + & 5 & = & 25 \end{array}$$

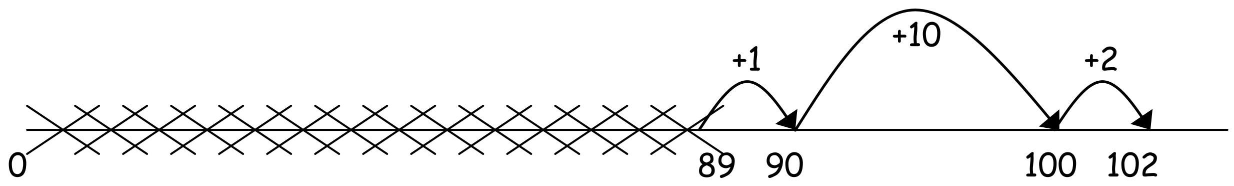
This would be recorded by the teacher as

$$\begin{array}{lrcrcl} & \overset{60}{\cancel{70}} & + & {}^{1}1 & & \\ - & \underline{40} & \underline{+} & \underline{6} & & \\ & 20 & + & 5 & = & 25 \end{array}$$

Children should know that units line up under units, tens under tens, and so on.

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

102 – 89 = 13



Partitioning and decomposition

In Year 3 children move on to subtracting HTU from HTU, the partitioning method can be demonstrated if required.

$$\begin{array}{lr} & 754 \quad = \\ - & \underline{86} \end{array}$$

$$\begin{array}{llrcrcrl} \text{Step 1} & & 700 & + & 50 & + & 4 & \\ & - & & & \underline{80} & \underline{+} & \underline{6} & \end{array}$$

$$\begin{array}{llrcrcrl} \text{Step 2} & & 700 & + & 40 & + & 14 & \textit{(adjust from T to U)} \\ & - & & & \underline{80} & \underline{+} & \underline{6} & \end{array}$$

$$\begin{array}{llrcrcrcl} \text{Step 3} & & 600 & + & 140 & + & 14 & & \textit{(adjust from H to T)} \\ & - & & & \underline{80} & \underline{+} & \underline{6} & & \\ & & 600 & + & 60 & + & 8 & = & 668 \end{array}$$

Formal written methods of column subtraction

Subtract numbers with up to 3 digits using the formal written methods of column subtraction where appropriate

```
  6 14 1
  7  5 4
-    8 6
  6  6 8
```

Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;*
- ✓ *know that decimal points should line up under each other.*

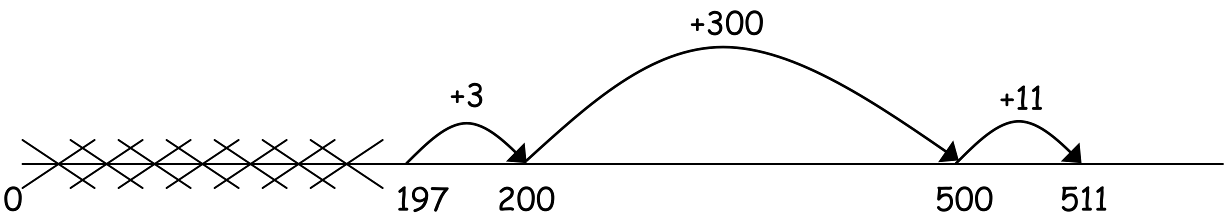
For example:

```
    7 1
  8.85
- 4.38
```

Alternatively, children can set the amounts to whole numbers, i.e. 895 – 438 and convert to pounds after the calculation.

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

511 – 197 = 314



Y4

Formal written methods of column subtraction

Subtract numbers with up to 4 digits

```
 614 1
 7545
-  862
 6683
```

In Years 5 and 6 the majority of children will be able to use the Year 3 and 4 strategies independently; consequently these methods will be applied to more complex problems. The teaching of these methods may be revisited at any time, as required.

Y5

Formal written methods of column subtraction - decomposition

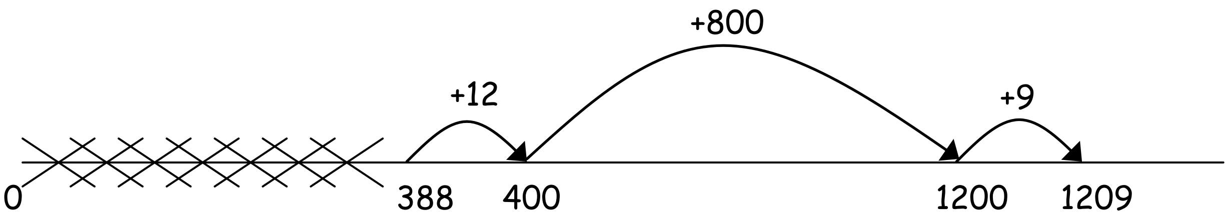
$$\begin{array}{r} \scriptstyle 6\,14\;1\;\;\; \\ \cancel{7}\cancel{5}47 \\ -\ \ 2861 \\ \hline 4686 \end{array}$$

Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;*
- ✓ *know that decimal points should line up under each other.*

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line may be a more appropriate method of subtraction.

1209 – 388 = 821



Y6

Formal written methods of column subtraction - decomposition

$$\begin{array}{r} \scriptstyle 5\,13\;1\;\;\; \\ \cancel{6}\cancel{4}67 \\ -\ \ 2684 \\ \hline 3783 \end{array}$$

Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other.*

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line may well be a more appropriate method.

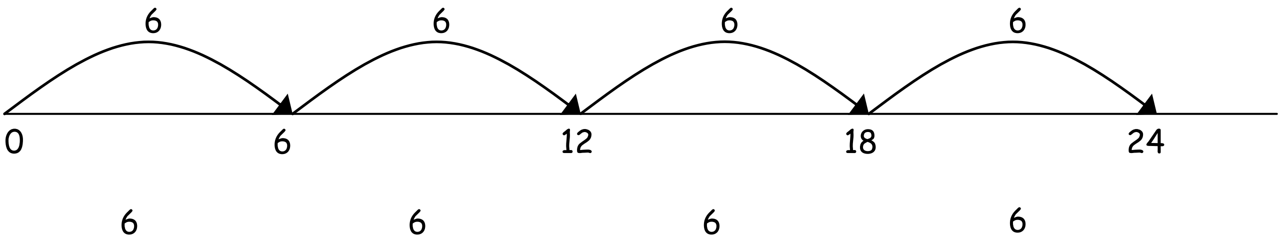
Y3

Children will be encouraged to see multiplication as:

✓ **Repeated addition**

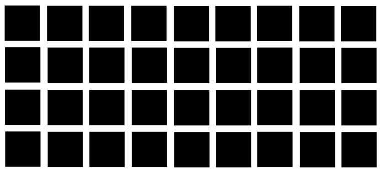
4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4

Children should use number lines or bead bars to support their understanding.



✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method. Counters and Cuisenaire rods are effective ways for children to build arrays.



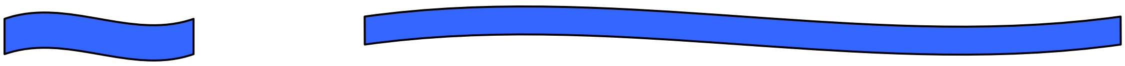
4 x 9 = 36

9 x 4 = 36

Children will also develop an understanding of

✓ **Scaling**

e.g. Find a ribbon that is 4 times as long as the blue ribbon



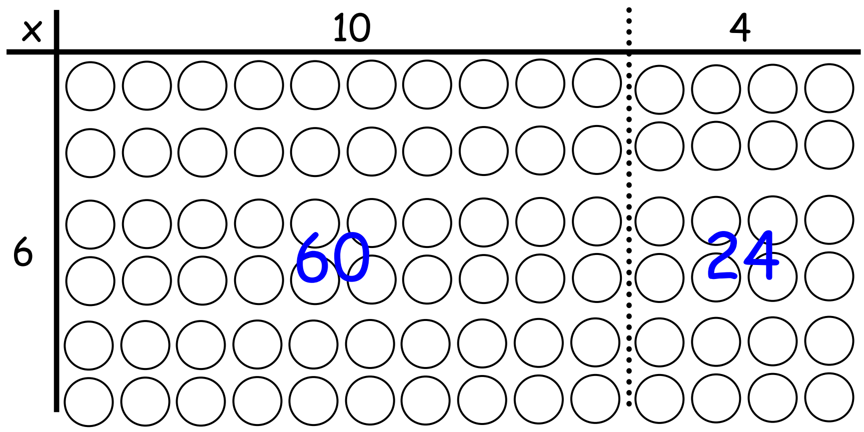
5 cm 20 cm

✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

□ x 5 = 20 3 x △ = 18 □ x O = 32

✓ **Children should know their 2x,10x,5x,3x,4x,8x tables**

Children will continue to use arrays where appropriate leading into the grid method of multiplication.
This method can then be modelled by the teacher using arrays.



(6 x 10) + (6 x 4)

60 + 24

84

Grid method

Children will approximate first
23 x 8 is approximately 25 x 8 = 200

x	20	3
8	160	24

$$\begin{array}{r} 160 \\ +\ \ 24 \\ \hline 184 \\ \hline \end{array}$$

Formal Written Layout Pupils who have mastered the grid method and demonstrate an in depth understanding of the principles that underpin it will be taught to multiply two-digit and three-digit numbers by a one-digit number using a formal written layout.

24 × 6 becomes

$$\begin{array}{r} 2\ 4 \\ \times\ \ \ \ 6 \\ \hline 1\ 4\ 4 \\ \hline {\scriptstyle 2}\ \ \end{array}$$

Answer: 144

342 × 7 becomes

$$\begin{array}{r} 3\ 4\ 2 \\ \times\ \ \ \ \ \ \ 7 \\ \hline 2\ 3\ 9\ 4 \\ \hline {\scriptstyle 2\ \ 1}\ \ \end{array}$$

Answer: 2394

Y4

Grid method

Children will approximate first

23 x 8 is approximately 25 x 8 = 200

x	20	3
8	160	24

```
  160
+  24
  184
```

Formal Written Layout – Pupils will be taught to multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

24 × 6 becomes

```
    2  4
 ×     6
 1  4  4
    2
```

Answer: 144

342 × 7 becomes

```
    3  4  2
 ×        7
 2  3  9  4
    2  1
```

Answer: 2394

- ✓ **Children should know their all their tables to 12x**

Y5

Grid method

ThHTU x U

(Short multiplication of up to 4 digits – multiplication by a single digit)

1346 x 9

Children will approximate first: 1346 x 9 is approximately 1350 x 10 = 13500

x	1000	300	40	6
9	9000	2700	360	54

```
   9000
   2700
+   360
+    54
  12114
   1 1
```

Formal Written Layout

Multiply numbers up to 4 digits by a one digit number using a formal written method.

2741 × 6 becomes

	2	7	4	1
×				6
1	6	4	4	6
	4	2		

Answer: 16 446

TU x TU

(Long multiplication – multiplication by more than a single digit)

72 x 38

Children will approximate first

72 x 38 is approximately 70 x 40 = 2800

x	70	2
30	2100	60
8	560	16

```
   2100
+   560
+    60
+    16
   2736
```

Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.

e.g. 4.9 x 3

Children will approximate first

4.9 x 3 is approximately 5 x 3 = 15

x	4	0.9
3	12	2.7

```
   12
+   2.7
   14.7
```

Formal Written Layout

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

```
   2 4
 x 1 6
 -----
   2 4  (6x4)
 1 2 0  (6x20)
   4 0  (10x4)
 2 0 0  (10x20)
 -----
 3 8 4
 -----
```

```
   2 4
 x 1 6
 -----
    2
 1 4 4  (6x24)
 2 4 0  (10x24)
 -----
 3 8 4
 -----
```

<u>Y6</u>

ThHTU x U

(Short multiplication – multiplication by a single digit)

4346 x 8

Children will approximate first

4346 x 8 is approximately 4346 x 10 = 43460

x	4000	300	40	6
8	32000	2400	320	48

```
  32000
+  2400
+   320
+    48
-------
  34768
```

Formal Written Layout

Multiply numbers up to 4 digits by a one digit number using a formal written method

2741 × 6 becomes

```
    2  7  4  1
 ×           6
 -------------
 1  6  4  4  6
 -------------
    4  2
```

Answer: 16 446

HTU x TU
(Long multiplication – multiplication by more than a single digit)

372 x 24

Children will approximate first
372 x 24 is approximately 400 x 25 = 10000

x	300	70	2
20	6000	1400	40
4	1200	280	8

```
  6000
+ 1400
+ 1200
+  280
+   40
+    8
  8928
   1
```

Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.

For example:

4.92 x 3

Children will approximate first
4.92 x 3 is approximately 5 x 3 = 15

x	4	0.9	0.02
3	12	2.7	0.06

```
  12
+  0.7
+  0.06
  12.76
```

Formal Written Layout

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

```
   1 2 4
x    5 6
--------
   1 2
   7 4 4
 1 2
 7 4 4 0
--------
 8 1 8 4
--------
 1
```

Division

Y3

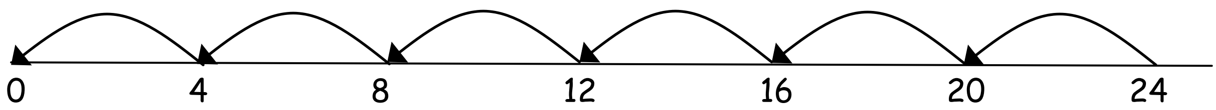
The children should be aware that numbers can be divided using either a sharing or grouping method. Ensure that the emphasis in Y3 is on grouping rather than sharing, as this is the basis for the chunking method (repeated subtraction).

Children will continue to use:

✓ **Repeated subtraction using a number line**

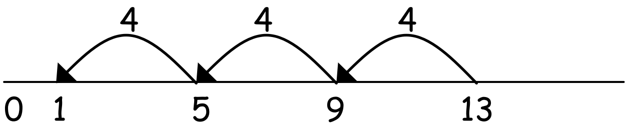
Children will use an empty number line to support their calculation.

24 ÷ 4 = 6



Children should also move onto calculations involving remainders.

13 ÷ 4 = 3 r 1



✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations**

26 ÷ 2 = □ 24 ÷ △ = 12 □ ÷ 10 = 8

✓ **Formal short division.** Where children are secure, an introduction to formal short division will be taught (no remainders).

98 ÷ 7 becomes

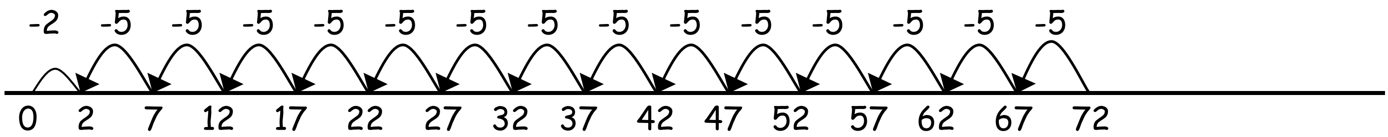
$$\begin{array}{r|ll} & \mathbf{1} & \mathbf{4} \\ \hline \mathbf{7} & \mathbf{9} & {}^{2}\mathbf{8} \end{array}$$

Answer: 14

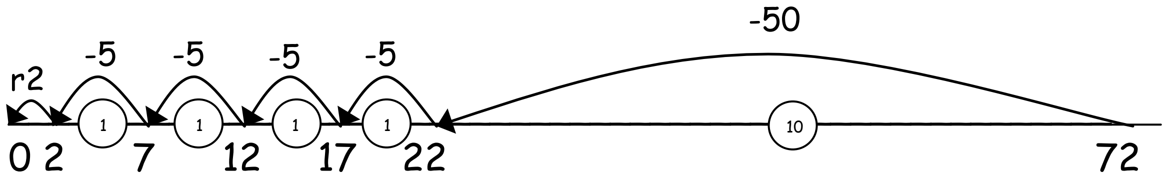
Y4

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar.

72 ÷ 5



Moving onto chunking on a number line:



Then onto the vertical chunking method:

Chunking TU ÷ U

72 ÷ 3

24	
3) 72	
- 30	10x
42	
- 30	10x
12	
- 6	2x
6	
- 6	2x
0	
Answer :	24

Leading to subtraction of other multiples.

96 ÷ 6

1 6	
6) 96	
- 60	10x
36	
- 36	6x
Answer :	16

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

e.g. I have 62p. Sweets are 8p each. How many can I buy?
Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?
Answer: 8 (the remaining 6 apples still need to be placed into a box)

✓ **Formal short division.**
Where children are secure, an introduction to formal short division will be taught (no remainders).

98 ÷ 7 becomes

$$\begin{array}{r|l} & 1\quad 4 \\ \hline 7 & 9\quad {}^{2}8 \end{array}$$

Answer: 14

Y5

Children will continue to use written methods to solve short division TU ÷ U.

Children can start to subtract larger multiples of the divisor, e.g. 30x

✓ **Chunking HTU ÷ U**

196 ÷ 6

```
       32 r 4
  6 )  196
     -  180      30x
         16
     -   12       2x
          4
```

Answer : 32 remainder 4 or 32 r 4

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example 240 ÷ 52 is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context.

✓ **Formal short division.** Formal short division will be taught (including interpreting remainders).

Short division

98 ÷ 7 becomes	432 ÷ 5 becomes	496 ÷ 11 becomes
$\begin{array}{r\|l} & 1\ \ 4 \\ \hline 7 & 9\ {}^{2}8 \end{array}$	$\begin{array}{r\|l} & 8\ \ 6\ \ r2 \\ \hline 5 & 4\ \ 3\ {}^{3}2 \end{array}$	$\begin{array}{r\|l} & 4\ \ 5\ \ r1 \\ \hline 1\ 1 & 4\ \ 9\ {}^{5}6 \end{array}$
Answer: 14	Answer: 86 remainder 2	Answer: $45\frac{1}{11}$

Y6

Children will continue to use written methods to solve long division TU ÷ U and HTU ÷ U.

✓ **Long division HTU ÷ TU**

972 ÷ 36

$$\begin{array}{rr l} & 27 & \\ \hline 36\ ) & 972 & \\ - & \underline{720} & 20\times \\ & 252 & \\ - & \underline{252} & 7\times \\ & 0 & \end{array}$$

Answer : 27

Any remainders should be shown as fractions, i.e. if the children were dividing 32 by 10, the answer should be shown as $3\ {}^{2}/_{10}$, which could then be written as $3\ {}^{1}/_{5}$ in it's lowest terms.

Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.

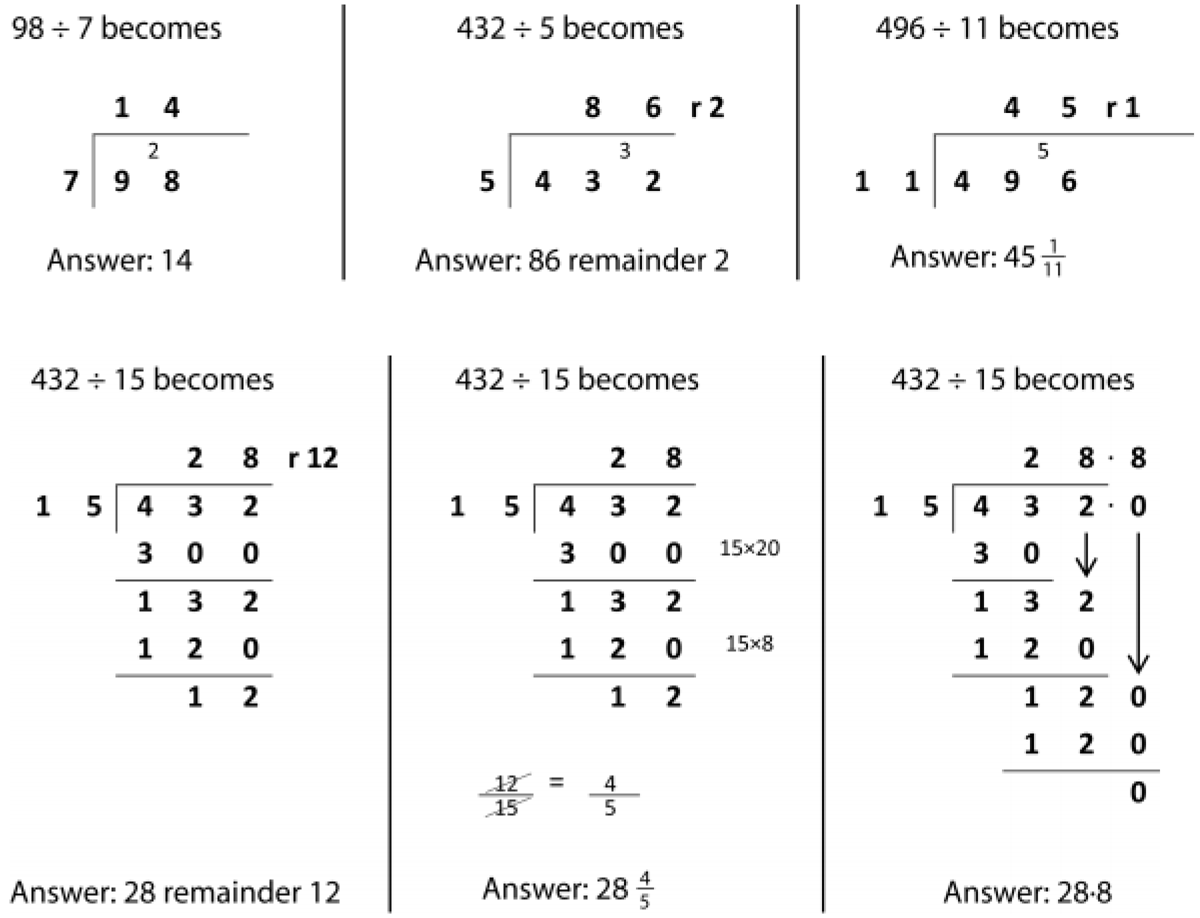
87.5 ÷ 7

$$\begin{array}{rr l} & 12.5 & \\ \hline 7\ ) & 87.5 & \\ - & \underline{70.0} & 10\times \\ & 17.5 & \\ - & \underline{14.0} & 2\times \\ & 3.5 & \\ - & \underline{3.5} & 0.5\times \\ & 0 & \end{array}$$

Answer : 12.5

✓ **Formal short and long division.** Formal short and long division of up to 4 digit numbers by 2 digit numbers will be taught (including interpreting remainders).

Short division



➢ For any further guidance, parents should consult the class teacher or maths leader

DE Nov 17